

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS
MATH2010D Advanced Calculus 2019-2020

Problem Set 9

1. Find the Taylor polynomial of degree 3 generated by $f(x, y)$ at the point $(0, 0)$ if $f(x, y) = e^{(x+\sin 2y)}$.
2. Find the Taylor polynomial of degree 6 generated by $f(x, y)$ at the point $(0, 0)$ if $f(x, y) = \ln(1 + x \sin y)$.
3. (Optional) Let $f(x, y) = e^{x+2y}$.
 - (a) Evaluate $\int_0^{1/2} \int_0^{1/2} f(x, y) dx dy$.
 - (b) i. Find the Taylor polynomial $P_2(x, y)$ of degree 2 generated by $f(x, y)$ at the point $(0, 0)$.
ii. Compute $\int_0^{1/2} \int_0^{1/2} P_2(x, y) dx dy$.
Is it a good approximation of the integral in (a)? Why?
4. Find the absolute maximum and minimum points of the functions on the given domains.
 - (a) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the triangle bounded by the lines $x = 0$, $y = 2$ and $y = 2x$ in the first quadrant.
 - (b) $f(x, y) = x^2 + xy + y^2 - 6x + 2$ on the rectangle bounded by the lines $x = 0$, $x = 5$, $y = -3$ and $y = 0$.
 - (c) $f(x, y) = xy$ on the region $D = \{(x, y) : x \geq 0, y \geq 0 \text{ and } x^2 + y^2 \leq 4\}$.
5. Among all triangles with vertices on a given circle, find those that have the largest area.